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Language and Mathematical Models: Quotient in LT and LL Research

Abstract

In this paper there are proposed two general models of a natural language construction which, we firmly believe, may result from the general way of development of any branch of Mathematics. The different steps of development of any language have been widely investigated within the frame of generative grammar but they are still to be defined and further refined and specified, as language is a natural phenomenon in perpetual evolution. Consequently, the mastery of different steps of this non-stopping process may primarily lead to a better comprehension of the parameters and the potential of the structures and finally allow us to reach reliable conclusions. Therefore, it is very important that, when mathematical models are used in LT and LL research, extra attention to be paid so that every step should be investigated for a complete development of the model. In present paper we focus on one of the suggested models, namely the Cartesian product and quotient procedure. The model is analysed and a number of applications in language teaching and learning with specific examples. The proposed model is within the scope of globalization of sciences; yet, our firm belief is that special characteristics should be preserved and the invariant elements should be consolidated.

Keywords: model, quotient, Cartesian product, projection, associativity, commutativity, mapping.

1. Introduction

Mathematicians trying to offer models to applied sciences normally focus on various aspects. In this effort some fields of mathematics were also introduced, as the *Category Theory* (MacLane, 1971). Other sciences ask for models from mathematics by specifying certain aspects, a quite extravagant situation of which is that of the mathematicians being asked to construct rather 'complicated and complex' models, as in Cryptography. Even more so, it is normal to order mathematical models for sciences that seem to have no connection with mathematics at first sight. Then mathematicians 'create' mathematics as the *Fuzzy Theory*, the *Chaos Theory or* the *Theory of Hyperstructures.* As for *linguistics,* it has always been associated with the use of mathematical models ever since it was first established as a science based on experiment and observation. Hjelmslev (1943) has considered language as a well-organized system of categories that not only can be analyzed and studied but also can be represented mathematically. Nevertheless, the demand for models applicable in linguistic theory has been more of a rush during the second half of 20th century with Chomsky's "mathematicalization" of the language. However, this interaction between linguistics and mathematics is not new. Mathematics "owe" to linguistics at least since Panini's times, 4th century BC. As R. Mankiewicz (2000) mentions, if Greek mathematics is based on philosophy, Indian mathematics is based on linguistics, and even more so on Panini's and the other great Indian linguists' work. As a conclusion, one could claim that mathematical models pouring into every field of applied sciences, including linguistics, might promote research development, provided they are appropriately used. M. Cross & A.O. Moscardini, point out:

> *"Furthermore, the motivation for modelling is that it provides a relatively cheap and rapid means of answering ill-posed questions concerning the system and the process". (Cross et al., 1985, p. 24, my underlinings)*

In this paper we propose a classification of the construction procedure and we point out some motivating examples from mathematics. Moreover, we claim that the proposed procedure does also exist in Linguistic Theory. The final implication is that some mathematical models may offer more than their creators intended to do.

2. Models of mathematical models

In the creation of a mathematical theory several general or specific methods, are used. However, in order to have a specific theory considered complete, one is expected to work through several stages of process, or 'steps'. Of course, all these steps should not be expected to be of equal length or of equal difficulty. In Vougiouklis et al (2000) there are suggested two general ways of development and study, applied in virtually every subject of mathematics. They are two procedures which are traced consciously or subconsciously, yet undeviatingly.

More specifically, these two procedures are as follows:

(I) First General Model

We recall that for a 'complete' study in mathematics, virtually in all branches, one could identify the following steps:

(i) The choice of the basic set of the study
(ii) Choice of the axioms, i.e. the basic rules of the construction
(iii) Construction
(iv) Morphisms, i.e. principal mappings transferring the structures or basic constructing elements.
(v) Endomorphisms, i.e. transformations and their characteristic, invariant, elements.

(II) Second General Model

Mathematicians believe that in Mathematics there are generally two inverse procedures:

(a) the product, called Cartesian product, which is a very simple procedure and is based on the ordering of the objects, and
(b) the quotient, which, by contrast to the product, is a very complicated procedure and not unique.

By following these steps in both general models, the exposition of a theory of Mathematics may be considered completed although more new constructions could be introduced and studied at every step.

The application of a certain structure as a model is an entirely different issue. Every applied science can occasionally use and -if not appropriate- reject mathematical models from every field of Mathematics; this by no means implies that the models are right or wrong but simply that they can be used or not for the specific purpose.

With present paper we propose the above models for further study within the scope of globalization of sciences as the Category Theory does. However, we persist on the step of the invariant elements because we respect all special characteristics that actually promote science and culture.

2.1. Language invariant as a uniform way of communication

Now let us try to elaborate on the five steps of the First General Model above:

In step (i), we have to specify the initial concepts and the set of elements to be studied. At this stage, it would be necessary to provide all possible elements to be used in every stage of application of the desired model.

In step (ii), we choose the appropriate axioms in order to build the structure, select the basic construction elements and establish the construction rules. These rules should be as limited - and appropriately selected - as possible so that they should not lead to inconsistencies, that is to say the destruction of the structure.

In the step (iii) of construction we form the structures and introduce new construction elements using proofs in every case. We test for identification of possible inconsistencies and if there are any, we return to step (ii) and redefine the axioms.

In step (iv), we define the morphisms which are the mappings transferring constructing elements from one structure to another or, more interesting, occasionally within a single structure. More specifically, by means of morphisms selected parts of a structure are transferred to another. This transferring may reveal similarities in structures which, at first sight, might have seemed to be different. In other words, at this step we study the 'motion' of structures. If we wish to explain it in terms of human senses this could be the actual transferring of a stimulus from one sense to another: For instance, *seeing* or *smelling* a rose may be 'pleasant', *feeling* or *tasting* it might prove to be rather 'unpleasant', though. Similarly, *"lemon tree's very pretty and the lemon flower's sweet but the fruit of the poor lemon is impossible to eat*". Even more so, two different realizations of the same sense may co-exist and simultaneously cancel each-other, as in the case of a jelly-fish, which is nice and smooth to *feel* but at the same time this *feeling* might cause you unbearable rash. At this step (iv), natural languages are ready to supply their users with the set of structures necessary to produce the '*Literature'* of each language.

Moreover, at this step, specific - but generally applied - mappings are investigated. An interesting example of these mappings are the *projections*, that is, any mapping f such that $f^2 = f$, or, in other words, if you apply twice, the result is the same as if you apply only once.

The concept of the *parameter* also appears here and plays a crucial role. Nevertheless parameters in special branches of mathematics may give a special meaning to some mappings. For example, projections may cancel some parameters in exactly the same way we 'lose' the property of height when we draw a ground-plan.

The final step (v) focuses on morphisms in the same structure such as symmetry, reduction and projection which are usually called transformations. Invariant elements stay unchanged under mappings and this is of great importance in the process of structure construction. Furthermore, the invariant elements are sub-structures of the corresponding structures. In terms of real-life experience, this could be the case of the buildings which have remained undamaged - i.e. the *invariant*- after an earthquake- the *transformation.* Or, in the case of a projection of a three-dimensional object on a plane, the invariant element is the plane itself as the property of height is actually 'lost' or cancelled.

Linguistics, as every other applied science, asks for mathematical models from every field of Mathematics. The model in quest may concern a specific language (syntax, grammar, lexis), or a model applicable to every language (universal language). Chomsky assumes that here the basic questions are principals and parameter. These two elements clearly belong to steps (i) and (iv) respectively. Moreover, the core language is in fact the characteristic or the invariant element, so we refer to step (v).

2.2. Quotient as a simple mathematical model

The Cartesian product is a very simple procedure and is based on the ordering. It can be applied in several cases of objects (grammar, syntax, lexis) or on more general classes as a general model applicable to every language (universal grammar). By contrast the quotient is a very complicated procedure and not unique. With present paper we do not claim to introduce a new model but to emphasize on the fact that the two steps-Second General Model- have to be taken in order that the introduction of a new model should be considered complete.

Chomskyan Universal Grammar as a system of subtheories is actually a procedure of a product. N. Chomsky (1986) assumes that here the basic questions are the principles and parameters. Similarly, when U. Eco (1995) considers Latin and Vulgata appearing in Dante independent languages, then the pursuit of the perfect language is a Cartesian procedure.

Although it might appear to be *metalanguage*, we propose a procedure of quotient:

" Using a Cartesian product of subtheories, find an expanded theory; then, using a quotient, find a new theory which will actually contain the subtheories". (Vougiouklis and Kambakis-Vougiouklis, 2000, p.490)

The product of classes in partitions is quite widely used in the linguistic theory (see Gross, M., 1972).

Based on the respective theory form Mathematics (Vougiouklis, Th., 1995), the following are suggested:

"...in a given structure any arbitrary partition could potentially maintain certain axioms or related weaker axioms and it is in the researcher's hand to identify them...". (Vougiouklis and Kambakis Vougiouklis, 2002, p.510)

If associativity (or commutativity) is valid, then, in a case of arbitrary partition, we obtain the so called *weak associativity* (respectively *weak commutativity)*. That is to say, there are class elements which connect these classes in some kind of associativity (respectively commutativity).

Here is an example from language, actually two partitions partially arbitrary:

(a) Consider the partition each class of which contains all possible synonym words. In this partition the majority of the classes of the words are *singleton*s, i.e. they consist of only one element, as the majority of the words have no synonyms. However, every partition in language is characterised by the synchronic occurrence of each item, i.e. a word may have had a synonym in the past or may have one in future, but it has not any at present.

(b) Furthermore, if we refer to an electronic lexicon, e.g. spelling-check in a computer, then the number of the elements of the majority of classes is greater because they also include all possible morphological realizations of each item such as tense, person, gender, number, case, etc (also compounds and derivatives).

An example of class- behaviour in the above partitions is the following:

Actually thorough study offers security

In the first partition, word classes could possibly be as follows:

actually	*thorough*	*study*	*offers*	*security*
really	*complete*	*research*	*gives*	*safety*
in fact	*detailed*		*provides*	
as a matter of fact	*exhaustive*		*supplies*	

By choosing different representatives from each class, one could obtain a number which reaches 4x4x2x4x2 = 256 possible combinations. Of course not all of them are appropriate because they may be not in use or they mean something different. Yet, from a communicative point of view, they have a value as they could maintain communication, especially of the 'foreign-talk' type.

The same example becomes even more complex and complicated in (b):

actually	*thorough*	*study*	*offers*	*security*
		studies	*offer*	
			offered	
really	*complete*	*research*	*gives*	*safety*
		researches	*give*	
			gave	
			given	
in fact	*detailed*		*provides*	
			provide	
			provided	
as a matter of fact	*exhaustive*		*supplies*	
			supply	
			supplied	

Above we have 4x4x4x13x2=1664 different possible combinations, not all of them plausible, of course.

3. Applications

(1) *Economy of space in newspapers.*

Let us suppose that we have to handle the difficult problem of economy of space in a newspaper. It is a convention that a gap at the end of a 'word' manifests the end of the specific word. This manifestation may yield the implication that we are dealing with twenty-`seven rather than twenty -six letters in the English alphabet, the twenty-seventh letter being a gap or an 'empty-space'. In mathematics, 'empty space' is symbolized by '0 (zero)' which is said to have first appeared as late as mid 300 AD. Such an ingenious use of gaps virtually leads to a quotient where we have as many subsets as the number of the letters consisting the longest possible word. Consequently, if we want to economize on space we should cut the '27th letter', that is the gap at the end of each word, out. This would lead to strings of letters without gaps amongst them. This practice was quite common amongst Ancient Greeks who wrote without gaps between words maintaining in this way a better correspondence between spoken and written form, as Bauer (1988) points out. Would this ever happen, we should automatically encounter another problem: how would the end of each word be indicated? Should we possibly 'invent' a set of final letters? Yet, such a solution would be against the basic principle of economy of language. A more plausible solution would possibly be to use a set of letters which do exist, yet they are less used and only in specific conditions, that is the list of capital letters. In this case our proposal could be formed as follows:

Abolish the empty space between words and indicate the beginning of each word using a capital letter

That is, in terms of our proposal:

AbolishTheEmptySpaceBetweenWordsAndIndicateTheBeginningOfEachWord UsingACapitalLetter

At this point one should mention that experimentation of this kind is justified and has been quite often used in newspapers, magazines and advertisements. Another solution could be to use different colours, the most convenient being black and gray, in turns, or any other colours provided that it would be both plausible and economic.

(2) *The partition of a written document.*

As we have already mentioned quotients are absolutely arbitrary partitions. Although this fact is hardly acceptable by the vast majority of people, its certain applications are often considered to be self-evident. Let us examine such a ‘self-evident’ application concerning the partition of a written document. A certain partition of a written document into sentences, paragraphs and chapters involves the author’s decision, who, apparently subconsciously, creates quotients whose contents refer to separate concept units. A further creation of arbitrary quotients takes place when s/he starts typing them. Now, the automatic change of line and page are undoubtedly not only arbitrary quotients but also temporary because they are bound to change with every adjustment of the top, bottom, left and right margins, line spacing, e.g. single spaced, double spaced etc, as well as the size and type of fonts to be used. Consequently, every set of words cannot and should not be characterised according to its content. Nevertheless, we do accept this fact to such extent that we do make chapter, page and line references, that is to say we accept the arbitrary as self-evident. Besides, the arbitrary characterises all languages according to Saussure and, before him, Aristotle. To conclude with this issue we would like to point out that alphabetical indexes are based on arbitrary layout.

(3) *The compilation of school grammars.*

Another interesting case of product-quotient process in language research concerns the compilation of a school grammar. An efficient school grammar should not only contain the most commonly used and instantly recognizable types by the majority of the native speakers of that specific language, but it also should reflect the so-called educated talk and writing. Needless to say that it should not contain elements from various dialects and idiolects, as it represents the so-called grammar of the *standard* language. We all know that speakers of a language have at their disposal a number of options to make concerning almost any linguistic element including grammar, form, meaning and pronunciation. That is to say, a specific language element can be expressed in usually more than one ways by different users. In English, for example, teachers of EFL insist that the only “correct” pronunciation of <other> is [΄ʌ δ ə], yet NSs of English may also include variations such as [΄ʌ v ə], [΄ɔ δ ə] and [΄ɔ δ e r] amongst their choices.[1] Similarly, in Greek, the plural of the perfect tense Paratatikos is given by the official school grammar as follows: 1st person *[δenó- maste]* , 2nd person *[δenó-saste],* 3rd person *[δέno-ndan]* . However the majority of speakers[2] in the north seem to prefer another form: 1st person *[δenó-mastan]*, 2nd person *[δenó-sastan]*. Moreover, a widely used form for 3rd person plural in many parts of Greece is [*δenó-ndousan]* and *[δenó-tan(e)]* while speakers of standard-non idiomatic Greek- in Peloponnesus prefer [*δenó-sande]*. Consequently the compiler of a school grammar is faced with the construction of appropriate quotients from a number of products created by use in every case. His decision is based on many factors both within and outside the linguistic system and it takes time and hard to work to come to the appropriate quotient for each case. Furthermore, he should be prepared to receive plenty of criticism and see his quotient almost instantly expanding into products by the users of the language. A very good example of excellent work with products, is the work of the Alexandrian Dionysius of Thrace, whose work *Τέχνη Γραμματική (Ars Grammatica)*

[1] *Of course, we are all aware of the fact that we need a homogeneous code in order to communicate but who can decide so easily which type is the most widely used and accepted in every instant?*

[2] We are talking about very widely used types and not at all dialectic or idiosyncratic.

gives a concise, yet comprehensive review of contemporary (1st cent. B.C.) Greek language, only in fifteen pages and twenty five paragraphs (Robins, 1967 and 1979).

Free variation is also a case of product and quotient process, to be discussed elsewhere, as it is synonymy, too.

(4) *Rule and exception.*

We left the case of *rule and exception* last, as we would like to emphasise on it because we think that this is the most interesting application together with the compilation of a grammar in (3) above.

It has already been stated that in the first step of the First General Model we lay down the basic set of the study. This procedure can be repeatedly applied, yet to a lesser extent, during the process of the study. Consequently, it is necessary to designate a specific set each time. When the multitude of the elements is small, then we could enter every single element separately. The vowels of the English alphabet are: A, E, I, O and U (Collins, 1987) and a full description of the set is mostly and usually the case. This full description will be referred to as *the rule*, from now on. A detailed observation of the case of *the rule* in every science could lead us to the following conclusion and axiom:

If the rule expresses the full set, it will be called an absolute rule; however, if it does not express the full set, then the issue of the exceptions emerges.

Exceptions are usually presented as a total, without taking into account the different manifestations and special characteristics of each exception or group of exceptions. In our opinion there are more than one types of exceptions, each with their own realizations and partly or completely different from the others. We attempted a categorization of the them and we identified three different types, three different *quotients:*

(a) The first type, which will be referred to as ***R+,*** manifests itself when the formulation of the description of the set includes more elements which should be taken into account. For example, *the orthographic representation of the English sound [tʃ]* is always made with the cluster <ch>, except in: (i) certain French words such as ‘chalet’, ‘champagne’, chandelier’, chauvinism’ etc, where it is pronounced as [ʃ], and (ii) Greek words such as ‘chameleon’, ‘chaos’, ‘character’, etc, where it is pronounced as [k].

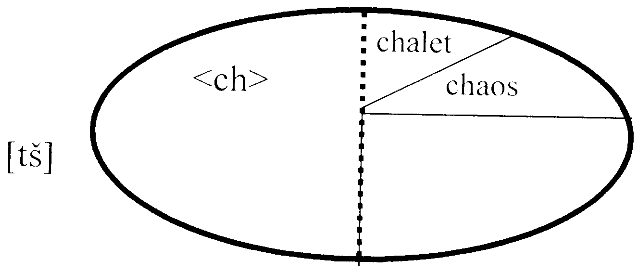

Figure 1

(b) The second type, which will be referred to as ***R-***, appears when the description does not cover the full set and, consequently, there are extra elements to be added in order to cover the full range: For example, the English sound *[z]* is represented by the letters <z>, and under certain circumstances, the letter <s>. Moreover, it is possible to be represented by the letter <x> in initial position, usually in Greek words such as <xylophone>, <xerography> etc.

(c)

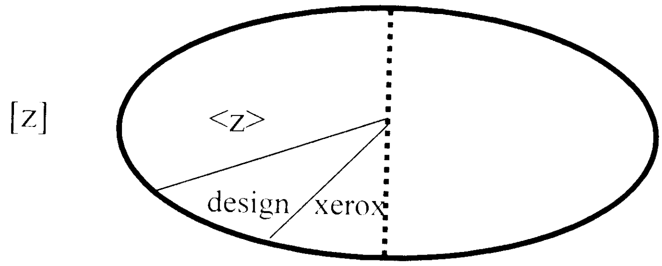

Figure 2

(d) The third type, which we will call ***R∓***, appears whenever the formulation of the description includes more elements to be taken into account but at the same time there are extra elements to be added in order to cover the full range. For example, in *English the sound [s] in initial or/and final position* is represented by the letter <s> at the beginning, and <ss> in the final position of a word; nevertheless the letter <s> could be also pronounced [ʃ] as in the word <sure>: this is the ***+ case*** in the ***R∓*** above. On the other hand, <ci> as in <cinema> and <civil>, and <ce> as in <center> and <certify> will be pronounced as [si] and [se] respectively and this is the ***– case*** in ***R∓*** above.

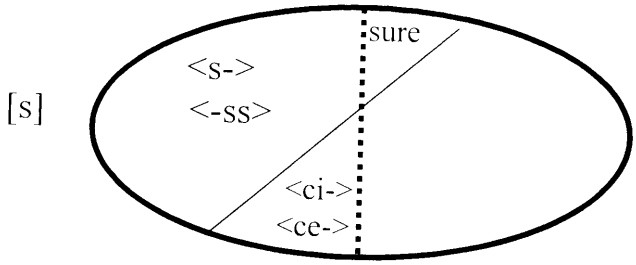

Figure 3

4. Teaching implications

(a) Both of the suggested General Models could be applied wherever we have to take ‘simple’, short steps: the bipole product-quotient, i.e. the Second General Model, or certain steps such as the use of invariant from the First General Model because, in this way, the learners can be trained to group, to line things up and to express in a uniform way small language problems. That is to say, they will be able to recognize certain language procedures they have already subconsciously mastered, or, in other words, to be language aware.

(b) The learners should also be encouraged to master the way of discovering the above mentioned ‘simple’ or small models, in such a way that they would be ready any moment to extract structures or rules in order to facilitate their own learning process. Needless to say, that these structures should not necessarily be learned by heart, provided the learners know how to reach them any moment, unless, of course, the learners decide they want to memorise a specific structure. With this proposal we want to indicate that in the teaching procedure we should not provide the learners with numerous sets of prefabricated rules to be memorised – and never used. By contrast we suggest that the learners should master the actual procedure of extracting the rules themselves, when needed.

In every case these 'simple' and 'small' models put forward our point of view that even in the procedure we believe that 'this is a wonderful world'.

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